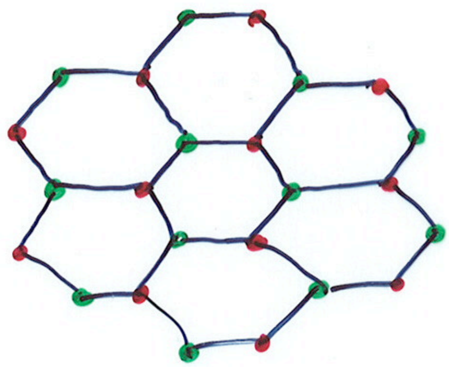
EMERGENT RELATIVITY IN GRAPHENE

IGOR HERBUT (SFU)

GRAPHENE : 2D CARBON ($1S_2$, $2S_2$, $2P_2$)



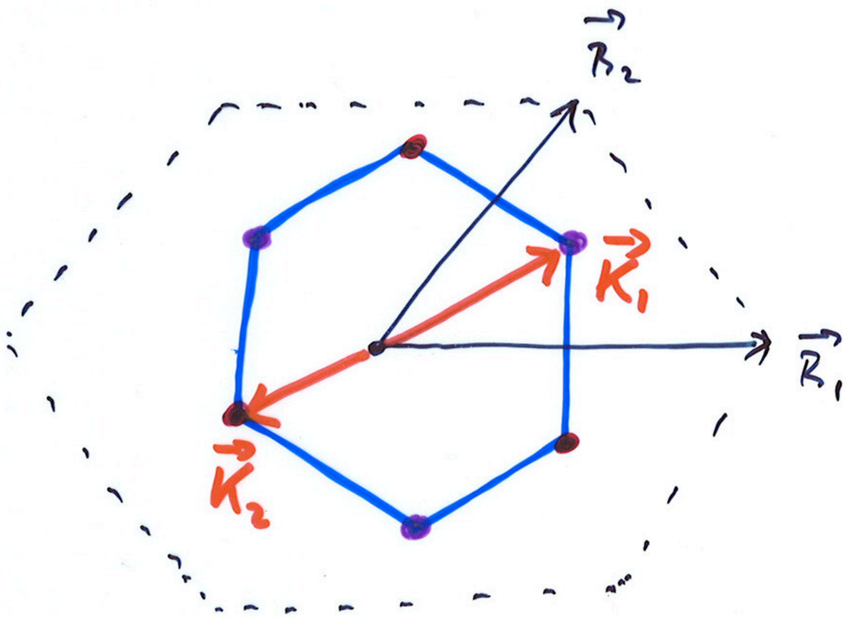
TWO SUBLATTICES:

• - A

• - B

ONE ELECTRON PER SITE FOR CONDUCTION!

RECIPROCAL LATTICE:



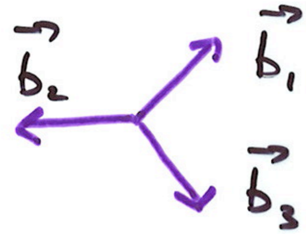
TIGHT-BINDING HAMILTONIAN: ($t \simeq 2.5\,eV$)

$$H = -t \sum_{A,\, b_i} U^{+}(\vec{A})\, V(\vec{A} + \vec{b}_i) + h.c.$$

RECIPROCAL SPACE:

$$H = -\int_{B.Z.} \frac{d^2\vec{k}}{(2\pi)^2} \left(U^+(\vec{k}), V^+(\vec{k}) \right) \begin{array}{c|c} 0 & t f(\vec{k}) \\ \hline t f^*(\vec{k}) & 0 \end{array} \begin{pmatrix} U^*(\vec{k}) \\ V(\vec{k}) \end{pmatrix}$$

COMPLEX: $f(\vec{k}) = \sum_{b_i} e^{i \vec{k}\cdot\vec{b}_i}$



ENERGY:

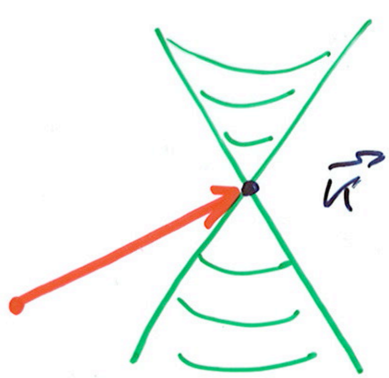
$$\varepsilon^2 = t^2 |f(\vec{k})|^2 = t^2 (\mathrm{Re}\, f)^2 + t^2 (\mathrm{Im}\, f)^2$$

AT $\mu = 0$, FERMI SURFACE AT $\varepsilon = 0$,

$$\mathrm{Re}\, f(\vec{k}) = \mathrm{Im}\, f(\vec{k}) = 0$$

$$\Downarrow$$

FERMI POINTS (AT $\vec{k} = \pm \vec{K}$)



"DIRAC CONE"
(ISOTROPIC)

NEAR DIRAC POINTS : $v \approx \frac{c}{300}$

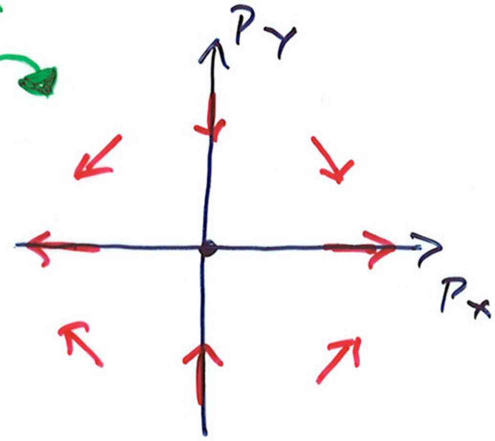
$$H = v \int d^2\vec{p}\,(U^+, V^+)\left[p_x \sigma_x - p_y \sigma_y\right]\begin{pmatrix} U \\ V \end{pmatrix}$$

$\vec{k} = +\vec{K} + \vec{p}$

$$+\, v \int d^2\vec{p}\,(U^+, V^+)\left[-p_x \sigma_x - p_y \sigma_y\right]\begin{pmatrix} U \\ V \end{pmatrix}$$

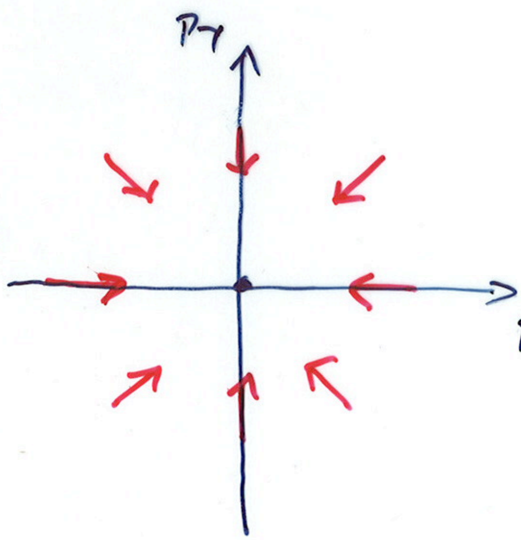
$\vec{k} = -\vec{K} + \vec{p}$

"VORTEX"



$+\vec{K}$

$-\vec{K}$



"ANTIVORTEX"

TOPOLOGY $\Rightarrow$ STABILITY !

PUT TWO POINTS TOGETHER:

$$H = \alpha_1 \hat{P}_x + \alpha_2 \hat{P}_y ,$$

$$\alpha_1 = \left(\begin{array}{c|c} \sigma_x & \\ \hline & -\sigma_x \end{array}\right) , \quad \alpha_2 = \left(\begin{array}{c|c} -\sigma_y & \\ \hline & -\sigma_y \end{array}\right) .$$

AND

$$\{\alpha_1, \alpha_2\} = \alpha_1 \alpha_2 + \alpha_2 \alpha_1 = 0 .$$

OR:

$$\alpha_1 = i \gamma_0 \gamma_1$$

$$\alpha_2 = i \gamma_0 \gamma_2$$

SO THAT

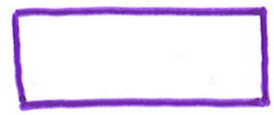
$$\{\gamma_\mu, \gamma_\nu\} = 2 \delta_{\mu\nu} \quad ; \mu = 0, 1, 2$$

(CLIFFORD ALGEBRA)

SYMMETRY:

I) EXACT

1) $\left.\begin{array}{c} A \leftrightarrow B \\ +K \leftrightarrow -K \end{array}\right\} D_2$



2) TIME-REVERSAL: $+K \leftrightarrow -K$
+
COMPLEX CONJUGATION

3) SPIN ROTATION

II) "EMERGENT"

1) LORENTZ

2) CHIRAL: $SU(2) = \{\gamma_3, \gamma_5, i\gamma_3\gamma_5\}$

WHERE $\{\gamma_3, \gamma_\mu\} = \{\gamma_5, \gamma_\mu\} = 0, \quad \mu = 0,1,2$

3) PARTICLE-HOLE: $\vec{M} = \{\gamma_0, i\gamma_0\gamma_3, i\gamma_0\gamma_5\}$

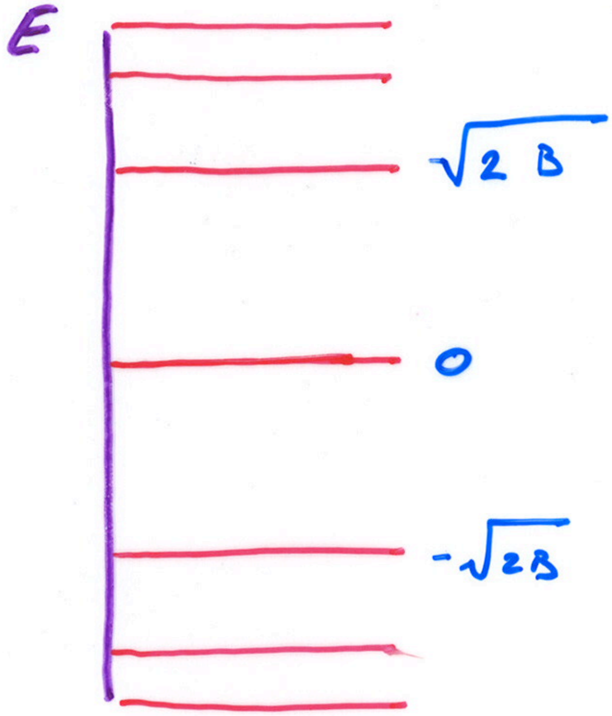
$$\Rightarrow \{\vec{M}, H\} = 0$$

$$M\psi_E = \psi_{-E}$$

MAGNETIC FIELD: $\ell \gg a$

$$H = i\gamma_0 \gamma_i \left(\hat{P}_i - eA_i \right) \quad ; \quad i = x, y$$

SPECTRUM: $E_n = \pm\sqrt{2n\, eB}$

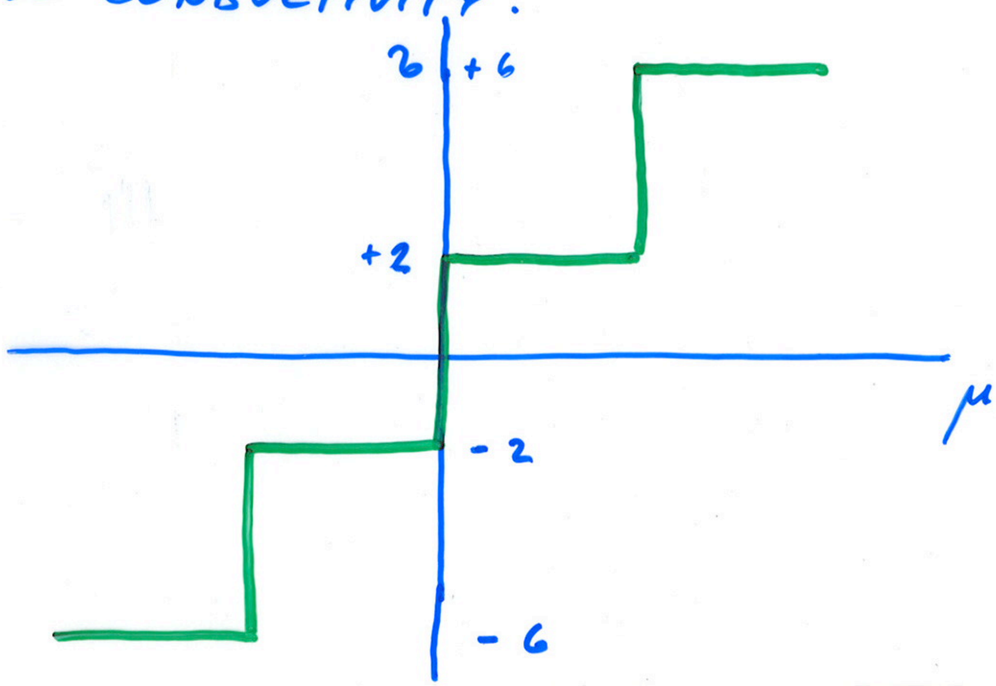


LANDAU LEVELS:

$$2\,(\text{spin}) \times \boxed{2\,(\text{Dirac})} \times \frac{eB}{hc} \cdot \text{Area}$$

DEGENERATE!

HALL CONDUCTIVITY:



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Y. Zhang et al, PRL '06 :

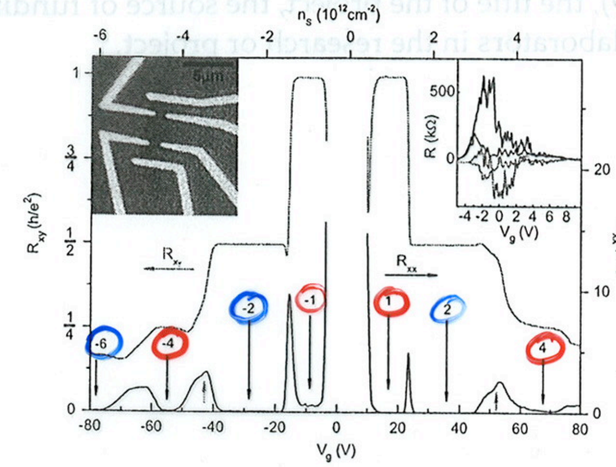


FIG. 1 (color online). R_{xx} and R_{xy} measured in the device shown in the left inset, as a function of V_g at $B = 45$ T and $T = 1.4$ K. $-R_{xy}$ is plotted for $V_g > 0$. The numbers with the vertical arrows indicate the corresponding filling factor ν. Gray arrows indicate developing QH states at $\nu = \pm 3$. n_s is the sheet carrier density derived from the geometrical consideration. Right inset: R_{xx} (dark solid lines) and R_{xy} (light solid lines) for another device measured at $B = 30$ T and $T = 1.4$ K in the region close to the Dirac point. Two sets of R_{xx} and R_{xy} are taken at different time under the same condition. Left inset: an optical microscope image of a graphene device used in this experiment.

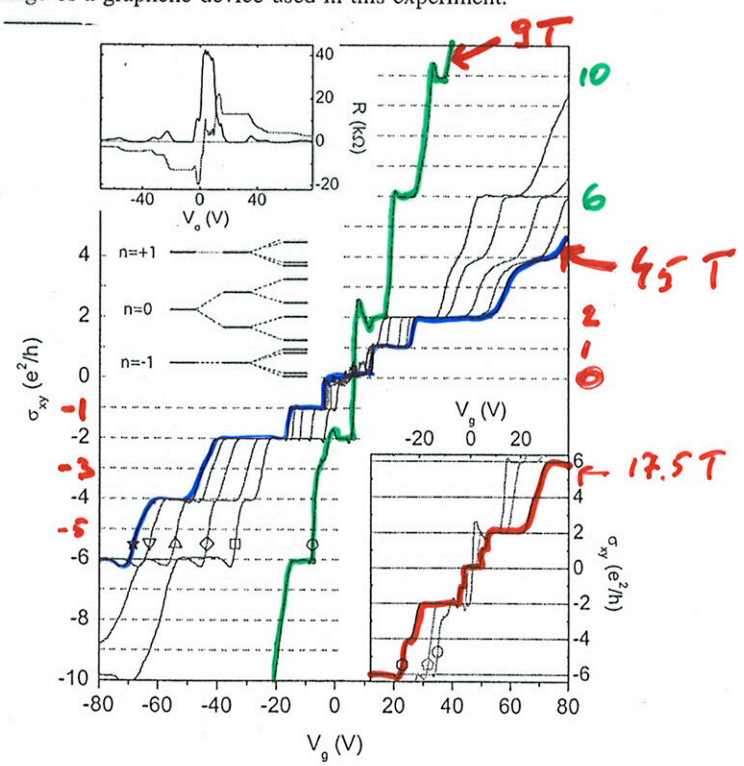


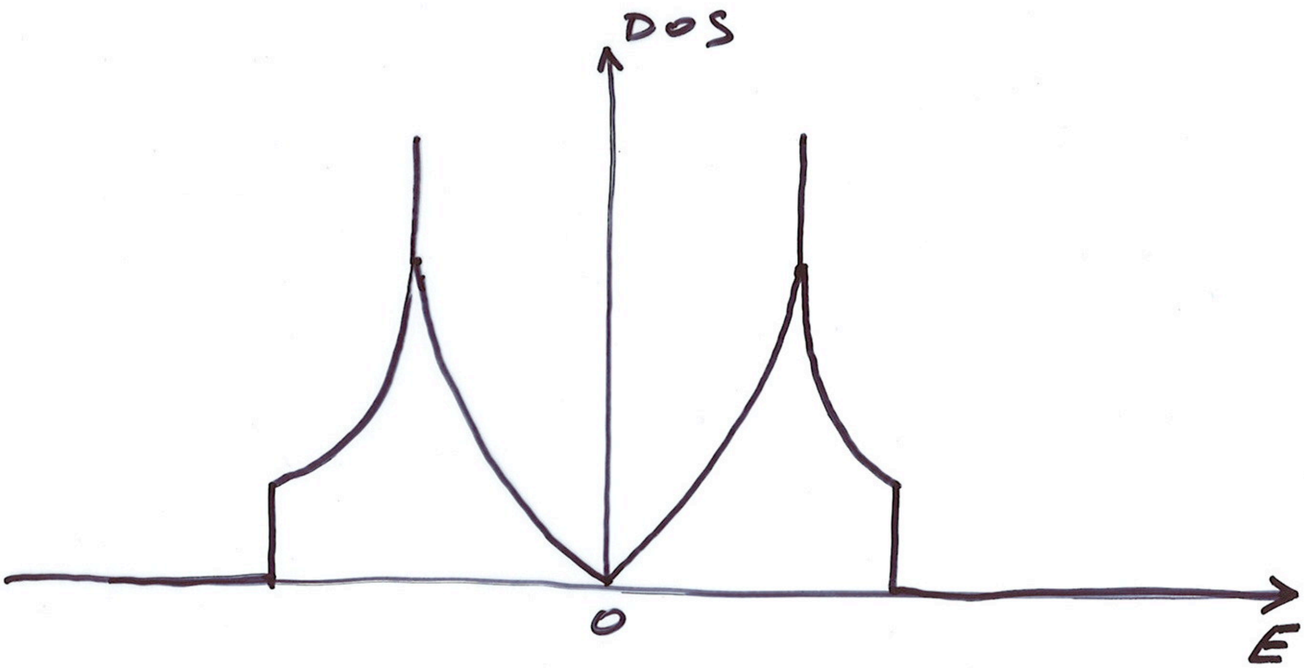
FIG. 2 (color online). σ_{xy}, as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$ T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{xy} for the same device measured at $B = 25$ T. Left lower inset: a sche-

INTERACTIONS?

COULOMB REPULSION : $g = \frac{2\pi e^2}{\varepsilon \hbar v_F} \approx 2$

ON-SITE REPULSION : $U \sim 10\,eV$

STRONG INTERACTION $\Longrightarrow$ GAP ("MASS")
($U_c / t \approx 5$)



WHICH GAP?

"MASS"

$$H = H_0 + mX \Rightarrow H^2 = H_0^2 + m^2$$

$$\Rightarrow \varepsilon = \pm\sqrt{p^2 + m^2}$$

IF :

$$\{X, H_0\} = 0 \quad \& \quad X^2 = 1 \ .$$

$$\Rightarrow \quad X = \vec{M} \in \{\gamma_0, \ i\gamma_0\gamma_3, \ i\gamma_0\gamma_5\} \quad \text{(CS)}$$

OR

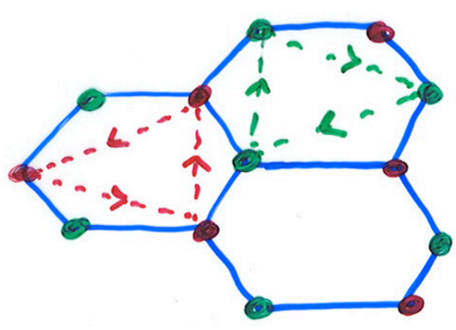
$$X = i\gamma_1\gamma_2 \quad \text{(TRS)}$$

ON LATTICE:

1) $\gamma_0 = \left(\begin{array}{c|c} \sigma_z & \\ \hline & \sigma_z \end{array}\right)$ – CDW, SDW

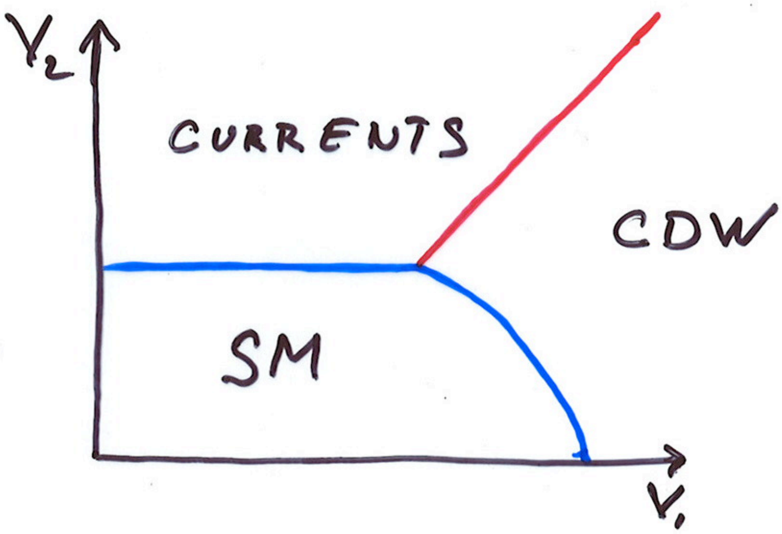
$a\, i\gamma_0\gamma_3 + b\, i\gamma_0\gamma_5$ – KEKULE

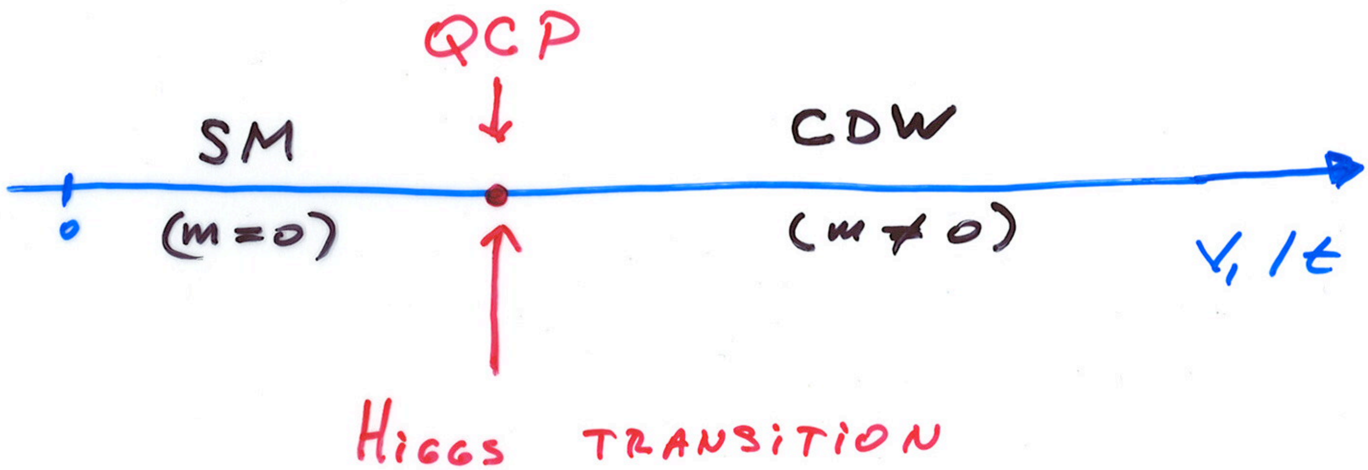
2) $i\gamma_1\gamma_2$ – CIRCULATING CURRENTS

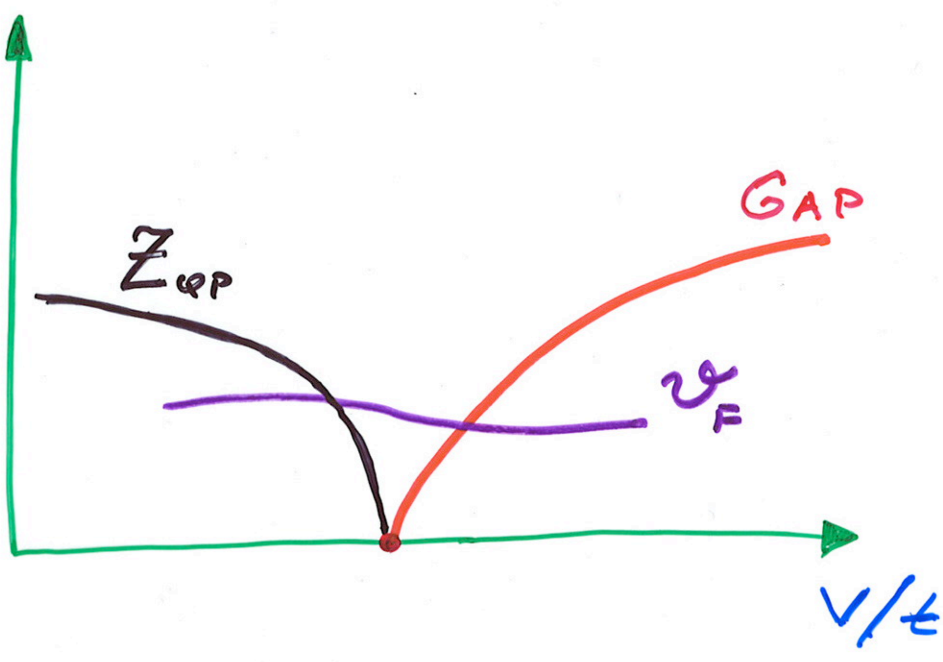


Simple model : V_1 - NN

V_2 - NNN







WRINKLES : $\xi \gg a$

OVERLAP ELEMENT "t" VARIES ! THE LATTICE HAMILTONIAN IS:

$$H = \begin{array}{c|c} & T \\ \hline T^{T} & \\ \underbrace{\qquad\quad}_{A} & \underbrace{\qquad\quad}_{B} \end{array}$$

DEFINE :

$$M = \begin{array}{c|c} I & \\ \hline & -I \end{array} \Rightarrow \{H, M\} = 0$$

SO WE HAVE :

1) TIME - REVERSAL
2) PARTICLE - HOLE SYMMETRY

WHAT IS THE LOW-ENERGY H ?

- $I_t = \left(\begin{array}{c|c} & I \\ \hline I & \end{array}\right) \cdot K$

NEAR DIRAC POINTS!

- $A \leftrightarrow B$ $\qquad \gamma_0 = \left(\begin{array}{c|c} \sigma_z & \\ \hline & \sigma_z \end{array}\right)$

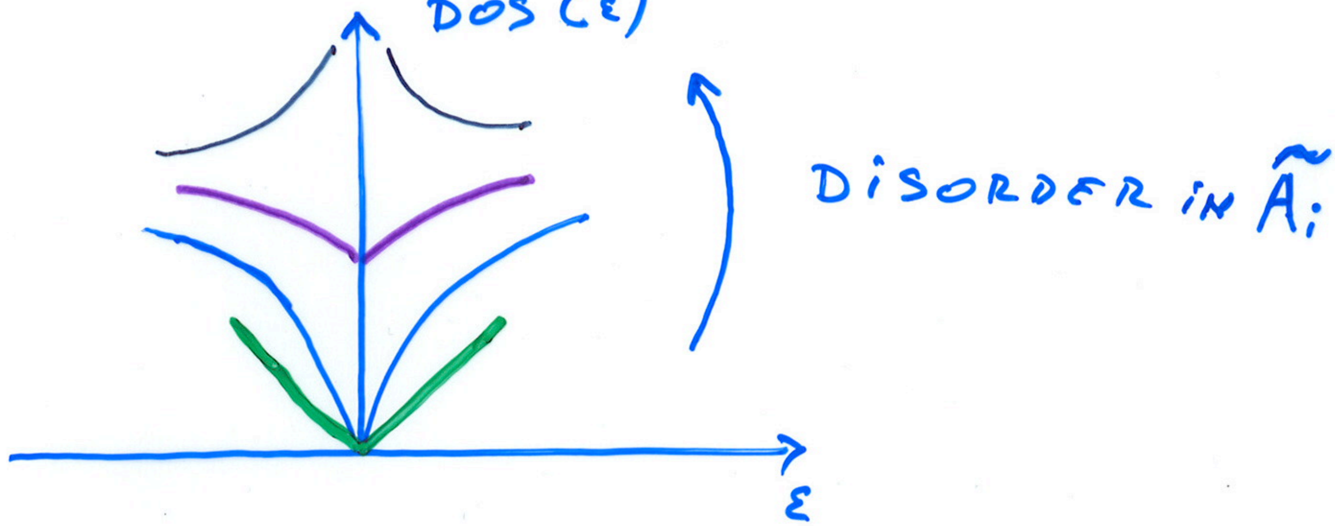
SO : $\qquad [H, I_t] = \{H, \gamma_0\} = 0, \quad H = H^+.$

UNIQUE ANSWER :

$$H = i\gamma_0\gamma_i(\hat{P}_i - i\gamma_3\gamma_5\tilde{A}_i) + m_1 i\gamma_0\gamma_3 + m_2 i\gamma_0\gamma_5$$

WHERE :

$$i\gamma_3\gamma_5 = \left(\begin{array}{c|c} I & \\ \hline & -I \end{array}\right).$$



WRINKLES ENHANCE INTERACTION EFFECTS!

SUM:

1) HONEYCOMB LATTICE ⇒ DIRAC EQ.

2) EMERGING: - RELATIVITY
- CHIRAL SYMMETRY
- PARTICLE-HOLE

3) QHE: STEPS AT EVEN INTEGERS

4) INTERACTIONS ⇒ GAP ("MASS")

5) WRINKLES ⇒ PSEUDO-MAGNETIC FIELD