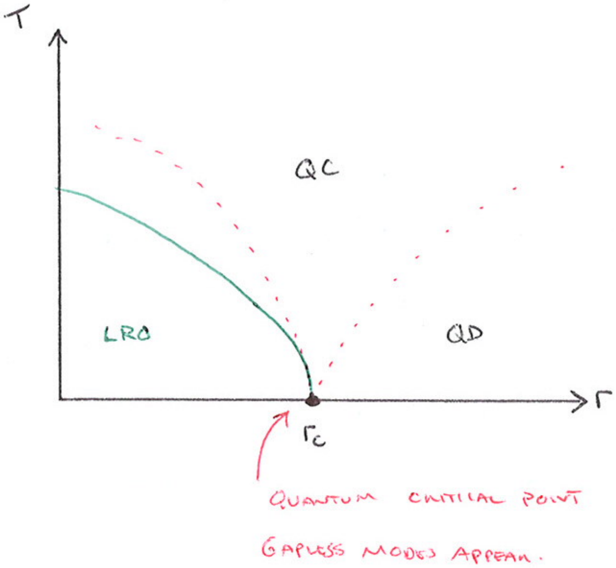
INTRODUCTION TO QUANTUM PHASE TRANSITIONS

05.27.2009

LECTURE I: SCALING & THE QUANTUM CLASSICAL MAPPING



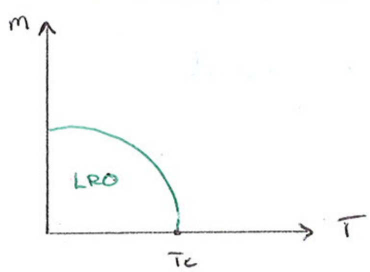
- THE GOAL OF THESE LECTURES IS TO UNDERSTAND & HAVE SOME INTUITION FOR THIS PHASE DIAGRAM

- Γ IS SOME NON-THERMAL TUNING PARAMETER. EXAMPLES INCLUDE:
  - ONSET OF SDW ORDER IN $CeCu_{6-x}Au_x$ (HEAVY FERMION MATERIAL) BY INCREASING x
  - LONG-RANGE MF-LIKE ORDER IN THE GEOMETRICALLY FRUSTRATED PYROCHLORE $Tb_2Ti_2O_7$ UNDER PRESSURE
  - ONSET OF FERROMAGNETISM BY TUNING MAGNETIC FIELD $LiHoF_4$.

- LOW ENERGIES MEAN LONG WAVELENGTHS, LEADING TO "GLOBAL" UNIVERSAL PHYSICS.

THE SCALING HYPOTHESIS

CONCERNED WITH CONTINUOUS (SECOND ORDER) PHASE TRANSITIONS WHERE THE ONSET OF SOME ORDER PARAMETER IMPLIES SYMMETRY BREAKING.



LANDAU THEORY ASSUMES THAT THE FREE ENERGY OF THE SYSTEM IS ANALYTIC IN THE SPATIALLY UNIFORM ORDER PARAMETER m & CAN BE WRITTEN AS THE EXPANSION:

$$F = hm + \overbrace{r_0(T-T_c)}^{r} m^2 + vm^3 + um^4 + \cdots$$

IF SYSTEM IS INVARIANT UNDER $m \to -m$ THEN $h = v = 0$. ONLY TEMPERATURE DEPENDENCE IS IN m^2 PREFACTOR.

$$\frac{\partial F}{\partial m} = 0 \Rightarrow m = \begin{cases} 0 & ; T > T_c \\ \sqrt{\dfrac{-r_0(T-T_c)}{2u}} & ; T < T_c \end{cases}$$

SO THE ONSET OF ORDER IS DESCRIBED BY: $m \sim |r|^\beta \quad , \beta = \frac{1}{2}$

MEAN FIELD CRITICAL EXPONENT.

CONT'D.

- THUS LANDAU THEORY WOULD PREDICT THIS BEHAVIOUR FOR ANY SYSTEM WITH $M \to -m$. HYPERUNIVERSALITY.

IN REALITY IT DEPENDS ON:
- DIMENSION OF SPACE
- SYMMETRY (# COMPONENTS) OF THE ORDER PARAMETER

- MAIN FLAW IN LANDAU THEORY IS THAT IT CANNOT ACCOUNT FOR FLUCTUATIONS IN THE ORDER PARAMETER WHICH ARE STRONGER AS d DECREASES OR N INCREASES. THUS WE HAVE:

d_{UCD}: DIMENSION ABOVE WHICH FLUCTUATIONS CAN BE NEGLECTED

d_{LCD}: DIMENSION BELOW WHICH FLUCTUATIONS ARE SO STRONG NO FINITE TEMPERATURE ORDER IS POSSIBLE.

$$d_{UCD} = 4 \quad \forall \text{ systems}$$

$$d_{LCD} = \begin{cases} 1 \; ; \; N = 1 \\ 2 \; ; \; N > 1 \end{cases}$$

$d = 2$, $N = 2$ IS THE SPECIAL MARGINAL CASE OF THE KT TRANSITION.

- FLUCTUATIONS CAN BE DEALT WITH IN THE LGW THEORY BY INTRODUCING A SPATIALLY DEPENDENT ORDER PARAMETER. THE CLASSICAL $O(N)$ MODEL IN d DIMENSIONS IS GIVEN BY:

COARSE GRAINED FIELD. AN AVERAGE OVER SOME REGION OF SPACE.

$$\mathcal{F}[\phi_a] = \int d^d x \left\{ |\nabla \phi_a(\vec{x})|^2 + \underbrace{r\,\phi_a^2(\vec{x})}_{\text{MASS TERM}} + \underbrace{\frac{u}{4!}\left[\phi_a^2(\vec{x})\right]^2}_{\text{SELF INTERACTION}} - \underbrace{h_a \phi_a(\vec{x})}_{\text{CONJUGATE FIELD.}} \right\}$$

WITH $\phi_a^2(\vec{x}) \equiv \phi_1^2(\vec{x}) + \cdots + \phi_N^2(\vec{x})$

PARTITION FUNCTION IS GIVEN BY:

$$Z = \int \mathcal{D}\phi_a \, e^{-\mathcal{F}[\phi_a]/k_B T} \quad \& \quad m \equiv \frac{1}{Z} \int \mathcal{D}\phi_a \, \phi_a \, e^{-\mathcal{F}[\phi_a]/k_B T}$$

CONTD.

RECALL THAT WE INTRODUCED $\mathcal{F}[\phi_a]$ TO STUDY THE SPATIAL FLUCTUATIONS OF ϕ_a WHICH ARE DEFINED BY ITS CORRELATION FUNCTION:

$$G(\vec{x}) = \langle \phi_a(\vec{x}) \phi_a(0) \rangle$$

WHICH IS EXPECTED TO DECAY EXPONETIALLY IN THE ORDERED PHASE.

$$G(\vec{x}) \sim e^{-|\vec{x}|/\xi}$$ CORRELATION LENGTH.

WITH $\xi \sim |r - r_c|^{-\nu}$, $\nu = \frac{1}{2}$ FOR THE GAUSSIAN THEORY.

THIS DIVERGING LENGTH SCALE HAS PROFOUND PHYSICAL IMPLICATIONS. IT WILL BE THE ONLY LENGTH SCAL AFFECTING PHYSICAL OBSERVABLES AS THE CRITICAL POINT IS APPROACHED. THIS IS THE CRUCIAL OBSERVATION IN THE SCALING HYPOTHESIS.

SUPPOSE WE RESCALE ALL LENGTHS (LATTICE SPACING, etc.) IN THE SYSTEM BY SOME POSITIVE VALUE, b, BUT ADJUST ALL EXTERNAL PARAMETERS $(T, h, ..)$ SUCH THAT ξ REMAINS UNCHANGED. THIS IS A SINGLE ITERATION OF THE RENORMALIZATION GROUP, & DUE TO THE SOLE DEPENDENCE ON ξ, ALL PHYSICAL QUANTITIES MUST REMAIN UNCHANGED.

FOR CONCRETENESS, CONSIDER THE FREE ENERGY DENSITY: $f = -\frac{K_B T}{V} \ln Z$

THE SINGULAR PART WILL BE A FUNCTION OF r & h $\quad f_s = f_s(r, h)$

PERFORMING THE PROCEDURE JUST DESCRIBED:

$$f_s(r,h) = b^{-d} f_s(\bar{r} b^{1/\nu}, \bar{h} b^{y_h}) \qquad \bar{r} = \frac{r}{r_0} \quad \bar{h} = \frac{h}{h_0}$$ RESCALED TO MAKE THEM DIMENSIONLESS.

TAKE $b = \bar{r}^{-\nu}$ THEN:
$$f_s(r,h) = \bar{r}^{d\nu} \Phi_f\left(\frac{\bar{h}}{\bar{r}^{\nu y_h}}\right)$$

MICROSCOPIC NON-UNIVERSAL DETAILS ARE IN h_0 & r_0. HOWEVER THE FORM OF Φ_f IS FULLY UNIVERSAL! THIS IS SCALING & IS A RESULT OF $\xi \to \infty$ AT THE CRITICAL POINT, LARGER & LARGER VOLUMES ARE AVERAGED OVER.

CONTD.

Quantum Statistical Mechanics

IN THE STUDY OF CLASSICAL PHASE TRANSITIONS WE WERE ABLE TO DEAL WITH A FREE ENERGY FUNCTIONAL $\mathcal{F}$, SINCE THE KINETIC & POTENTIAL PARTS OF THE HAMILTONIAN COMMUTED i.e. IF WE WRITE OUR HAMILTONIAN USING PHASE SPACE VARIABLES FOR N DEGREES OF FREEDOM,

$$\mathcal{Z} = \frac{1}{N!\,h^{Nd}} \int \prod_i d^d q_i \, d^d p_i \; e^{-\beta \mathcal{H}(\vec{p}_i, \vec{q}_i)}$$

$$= \frac{1}{N!\,h^{Nd}} \int \prod_i d^d p_i \; e^{-\beta \mathcal{H}_{KIN}(\vec{p}_i)} \int \prod_i d^d q_i \; e^{-\beta \mathcal{H}_{POT}(\vec{q}_i)}$$

$$= \mathcal{Z}_{KIN}\, \mathcal{Z}_{POT}$$

JUST A PRODUCT OF GAUSSIAN INTEGRALS & IS THUS SINGULARITY FREE.

WE CANNOT DO THIS IN THE QM CASE SINCE $[\mathcal{H}_{KIN}, \mathcal{H}_{POT}] \neq 0$ & INSTEAD WE MUST EVALUATE:

$$\mathcal{Z} = \mathrm{Tr}\, e^{-\beta \mathcal{H}}.$$

THIS EXPRESSION CAN BE ELEGANTLY ANALYZED IN FEYNMAN'S PATH INTEGRAL FORMULATION WHERE WE NOTE THAT $e^{-\beta \mathcal{H}}$ LOOKS LIKE THE USUAL TIME EVOLUTION OPERATOR OF QM WITH

$$\mathcal{U}(t) = e^{-\frac{it}{\hbar}\mathcal{H}}$$

BUT IN IMAGINARY TIME. i.e. $t = -i\beta\hbar$. WRITE THE TRACE AS A SUM OVER STATES

$$\mathcal{Z} = \sum_n \langle n | \mathcal{U}(-i\beta\hbar) | n \rangle$$

★ PARTITION FUNCTION IS JUST A SUM OF IMAG. TIME TRANSITION AMPLITUDES WHERE THE SYSTEM BEGINS & RETURNS TO THE SAME STATE $|n\rangle$.

WE ARE THUS LED TO THE FUNDAMENTAL CONCLUSION OF QUANTUM STAT MECH:

CONTD.

- CALCULATING THE THERMODYNAMICS OF A QUANTUM SYSTEM IS EQUIVALENT TO COMPUTING TRANSITION AMPLITUDES FOR ITS EVOLUTION OVER AN IMAGINARY TIME INTERVAL SET BY THE MEASUREMENT TEMP.

- IN PRINCIPLE ONE EVALUATES AN AMPLITUDE BY SUMMING OVER ALL POSSIBLE SPACE TIME TRAJECTORIES THAT CONNECT THE INITIAL AND FINAL STATE. THIS CANNOT BE DONE IN PRACTICE, BUT CAN BE DONE PERTURBATIVELY OVER AN INFINTESSIMAL TIME INTERVAL.

BREAK THE FULL TIME INTERVAL INTO M STEPS:

$$\mathcal{U}(-i\beta\hbar) = \left(e^{-\Delta\tau \mathcal{H}/\hbar}\right)^M = [\mathcal{U}(-i\Delta\tau)]^M$$

WHERE $\Delta\tau$ IS THE MICROSCOPIC TIME INTERVAL SUCH THAT: $\Delta\tau = \frac{\hbar\beta}{M}$

RETURNING TO OUR PARTITION FUNCTION, WE INSERT M COMPLETE SETS OF INTERMEDIATE STATES:

$$Z = \sum_n \langle n | [\mathcal{U}(-i\Delta\tau)]^M | n \rangle$$

$$= \sum_{n, m_1, \ldots, m_M} \langle n | e^{-\Delta\tau \mathcal{H}/\hbar} | m_1 \rangle \langle m_1 | e^{-\Delta\tau \mathcal{H}/\hbar} | m_2 \rangle \times \cdots \times \langle m_M | e^{-\Delta\tau \mathcal{H}/\hbar} | n \rangle$$

PHYSICAL INTERPRETATION IS A SUM OVER TRANSFER MATRICES PROVIDED WE ASSOCIATE THE IMAGINARY TIME DIRECTION WITH ANOTHER SPATIAL DIMENSION.

I.e. THIS LOOKS LIKE THE PARTITION FUNCTION FOR A CLASSICAL SYSTEM IN $d+1$ DIMENSION WHERE THE $+1^{th}$ DIMENSION IS NOT OF INFINITE EXTENT!

TAKING THE $M \to \infty$ $(\Delta\tau \to 0)$ LIMIT, THE SUM OVER STATES CAN BE CONVERTED INTO THE USUAL PATH INTEGRAL:

$$Z = \int \mathcal{D}\phi_a \, e^{-S[\phi_a]/\hbar}$$

FOR THE SIMPLE GLW THEORY PRESENTED ABOVE, THE FREE ENERGY FUNCTIONAL HAS BEEN PROMOTED TO AN ACTION:

TUNING PARAMETER

$$S[\phi_a] = \int_0^{\hbar\beta} d\tau \int d^d x \left\{ \frac{1}{2} \left[[\partial_\tau \phi_a(\vec{x},\tau)]^2 + c\,|\vec{\nabla}\phi_a(\vec{x})|^2 + r\phi_a^2(x,\tau) + \frac{u}{4!}[\phi_a^2(\vec{x})]^2 \right] \right\}$$

POINT OUT bare vs. renormalized

$$\phi_a(\vec{x}, \tau + \hbar\beta) = \phi_a(\vec{x}, \tau)$$

CONTD.

LETTING $T \to 0$, $\hbar\beta \to \infty$ & THE $d+1$ DIMENSIONAL CLASSICAL ANALOGUE BECOMES EXACT.

$S[\phi_a]$ DEFINES A CONTINUUM QUANTUM FIELD THEORY, WHICH BY ALTERING THE VALUE OF r @ $T=0$ CAN BE TUNED ACROSS A QUANTUM PHASE TRANSITION BETWEEN A STATE WITH $\langle \phi_a \rangle = 0$ & ONE WITH $\langle \phi_a \rangle \neq 0$

$$T=0 \quad \underset{\langle \phi_a \rangle \neq 0 \quad r_c \quad \langle \phi_a \rangle = 0}{\longrightarrow} \; r$$

EVERY SECOND ORDER QPT WILL HAVE SUCH A QFT DESCRIPTION, WITH (AT $T>0$) THE INTERESTING FEATURE OF IT BEING DEFINED ON A PECULIAR "SLAB" GEOMETRY WITH INFINITE EXTENT IN d SPATIAL DIMENSIONS BUT FINITE EXTENT IN THE IMAGINARY TIME DIRECTION

$$L_\tau = \hbar\beta = \frac{\hbar}{K_B T} \qquad \text{THERMAL LENGTH.}$$

MUCH OF WHAT WE KNOW ABOUT CLASSICAL PHASE TRANSITIONS CARRIES OVER, SUCH AS UNIVERSALITY (NEXT LECTURE). RECALL THERE THAT EVERYTHING WAS CONTROLLED BY A DIVERGING CORRELATION LENGTH:

$$\xi \sim |r - r_c|^{-\nu} \qquad \left(\nu = \tfrac{1}{2} \text{ FOR } u = 0\right)$$

NOW THAT WE HAVE DYNAMICS, CAUSALITY REQUIRES US TO INTRODUCE A "TEMPORAL" CORRELATION LENGTH (OR TIME) ξ_τ. IN $S[\phi_a]$ SPACE & TIME ENTERED THE ACTION IN THE SAME WAY. THIS IS KNOWN AS A RELATIVISTIC FIELD THEORY BUT IT NEED NOT BE THE CASE. IN GENERAL:

$$\xi_\tau \sim \xi^{z}$$

DYNAMIC CRITICAL EXPONENT.

Quantum - Classical Mapping:

- IMAG. TIME CORRELATIONS OF A d-DIMENSIONAL QUANTUM SYSTEM AT T ARE RELATED TO THE CORRELATIONS OF A $d+1$ DIMENSIONAL CLASSICAL SYSTEM WITH A FINITE EXTENT $L_\tau = \hbar / k_B T$ IN ONE DIMENSION.

THE DIVERGING TIME SCALE IS RELATED TO AN ENERGY SCALE $\hbar\omega = \hbar/\xi_\tau$ & WE MUST CONCLUDE THAT AT THE QCP $\omega_c = 0$. (CRITICAL FLUCTUATIONS).

i.e. A QUANTUM SYSTEM WILL BEHAVE CLASSICALLY PROVIDED $\hbar\omega \ll K_B T$

ALL MODES ARE FULLY POPULATED & THE ASYMPTOTIC CRITICAL BEHAVIOUR OF ANY TRANSITION WHICH OCCURS AT $T>0$ WILL BE STRICTLY CLASSICAL.

Introduction to Quantum Phase Transitions

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Lecture II: Quantum Critical Phenomena

- Recall that we found that if $\hbar\omega \ll k_B T$, the asymptotic critical behaviour will remain classical.

The Quantum Critical Point

- At $T=0$, the quantum phase transition occurs at the point where the characteristic energy scale of fluctuations above the ground state vanishes as $r \to r_c$.

- Let $\Delta = \hbar\omega$ be the energy gap away from criticality.

We know: $\xi \sim |r-r_c|^{-\nu}$; $\omega \sim \xi_\tau^{-1} \sim \xi^{-z}$

so $\Delta \sim |r-r_c|^{\nu z}$

- An important result will now become immediatey clear:

The presence of a gap indicates that any autocorrelation function should decay exponentially to zero at long times.

Consider the Heisenberg representation of any operator $\mathcal{O}$ in imaginary time:

$$\mathcal{O}(\tau) = e^{\mathcal{H}\tau/\hbar}\,\mathcal{O}\,e^{-\mathcal{H}\tau/\hbar}$$

Then, its auto-correlation function is:

$$\begin{aligned}
G_{\mathcal{O}}(\tau) &= \langle 0|\mathcal{O}(\tau)\mathcal{O}(0)|0\rangle \\
&= \langle 0|e^{\mathcal{H}\tau/\hbar}\mathcal{O}e^{-\mathcal{H}\tau/\hbar}\mathcal{O}|0\rangle \\
&= \sum_n \langle 0|e^{\mathcal{H}\tau/\hbar}\mathcal{O}|n\rangle\langle n|e^{-\mathcal{H}\tau/\hbar}\mathcal{O}|0\rangle \\
&= \sum_n e^{-(E_n-E_0)\tau/\hbar}\,|\langle 0|\mathcal{O}|n\rangle|^2
\end{aligned}$$

CONTD.

NOW, AS $\tau \to \infty$, ONLY THE $n=1$ TERM WITH $\Delta = \epsilon_1 - \epsilon_0$ WILL CONTRIBUTE SO:

$$G_{\mathcal{O}}(\tau) \sim e^{-(\Delta/\hbar)\tau} \sim e^{-\tau/\xi_c}$$

ALL CRITICAL SYSTEMS ARE GAPLESS!

- AWAY FROM CRITICALITY WE NOW HAVE 2 LARGE, BUT NOT INFINITE LENGTH SCALES, ξ & ξ_c, WHERE $|r-r_c| \ll 1$. WE MAY REPEAT OUR PREVIOUS HOMOGENEITY ARGUMENTS TO DERIVE A DYNAMIC SCALING FORM FOR ANY OPERATOR $\mathcal{O}$ WITH SCALING DIMENSION $\dim[\mathcal{O}]$:

$$\mathcal{O}(k,\omega,r) = \xi^{\dim[\mathcal{O}]} \Phi_{\mathcal{O}}(k\xi, \omega\xi_c)$$

$|r-r_c|$ ENCODED IN HERE

SCALING DIMENSION JUST ENCODES HOW AN OPERATOR CHANGES UNDER A RG TRANSFORMATION. OFTEN CLOSE TO THE ENGINEERING DIMENSION.

- DIRECTLY AT THE CRITICAL POINT, WE HAVE SAID THAT BOTH $\xi \to \infty$ & $\xi_c \to \infty$ SO THIS SCALING FORM NO LONGER MAKES SENSE. AT A SCALE INVARIANT CRITICAL POINT, THE ONLY MEANINGFUL LENGTH SCALE IS THE INVERSE OF THE WAVE VECTOR AT WHICH THE SYSTEM IS BEING PROBED. THIS SETS THE CHARACTERISTIC FREQUENCY AT:

$$\omega \sim k^z.$$

$$\mathcal{O}(k,\omega,r) \sim k^{-\dim[\mathcal{O}]} \Phi_{c\mathcal{O}}\left(\frac{\omega}{c k^z}\right)$$

SCALING DIMENSION $\mathcal{O}$ ENG. DIM. DEPENDS ON z.

- AT A CPT, A UNIQUE GROUND STATE IS SELECTED AT $T \to 0$. AT A QPT, FLUCTUATIONS ARE DRIVEN BY HEISENBERG UNCERTAINTY & PERSIST AT ALL LENGTH AND TIME SCALES. THESE FLUCS. HAVE CHARACTERISTIC FREQUENCY k^z

ALL COLLECTIVE MODES ARE OVERDAMPED $\Rightarrow$ INCOHERENT DIFFUSIVE REGIME.

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CONTD.

FINITE TEMPERATURE CROSSOVERS

- WE HAVE ALREADY LEARNED THAT OUR d-DIMENSIONAL CQFT CAN BE THOUGHT OF AS A $(d+1)$-DIMENSIONAL CFT HAVING INFINITE EXTENT IN THE d SPATIAL DIMENSIONS & FINITE EXTENT

$$L_\tau = \frac{\hbar}{k_B T}$$

IN THE IMAGINARY TIME DIRECTION.

- THE LEADING ORDER EFFECTS SHOULD BE ABLE TO BE DETERMINED BY APPEALING TO THE THEORY OF FINITE-SIZE-SCALING. BEFORE DOING THAT, WE LIST TWO POSSIBLE OUTCOMES OF INTRODUCING BOUNDARY CONDITIONS INTO OUR SYSTEM:

(i) THE TRANSITION IS COMPLETELY DESTROYED $\forall\ T>0$.

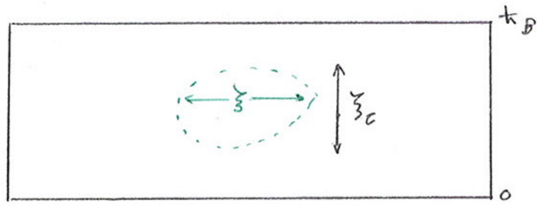
eg. ARRAY OF 1-d JOSEPHSON JUNCTIONS. QCM GIVES US A CLASSICAL $O(2)$ MODEL ON A STRIP OF WIDTH $c\hbar\beta$. BELOW d_{LCD} ~~XXXXXXX~~

(ii) THE TRANSITION PERSISTS AT $T>0$ BUT CHANGES UNIVERSALITY CLASSES.

eg. QUANTUM $O(2)$ MODEL IN $d=2$ IS A Cl. 3D XY MODEL @ $T=0$. AT $T>0$ THE SYSTEM IS MARGINAL & HAS A KT TRANSITION.

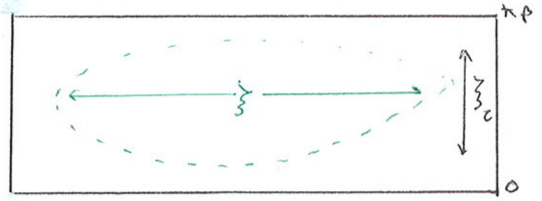
- THIS SEEMS DRASTIC, BUT IF WE ARE CLOSE ENOUGH TO THE QPT, THE CROSSOVER PHYSICS IS COMPLETELY CONTROLLED BY THE QCP. i.e. WE CAN CTSLY TUNE THE SYSTEM AWAY FROM CRITICALITY, NOT BY CHANGING A PARAMETER, BUT BY SMOOTHLY REDUCING ITS DIMENSIONALITY.

- IF THIS CAN BE DONE CONTINUOUSLY, THERE MUST BE SOME TEMP. SCALE WHERE FOR SOME VALUE OF r, $\xi_\tau \to L_\tau$, THE SYSTEM NOW REALIZES FOR THE FIRST TIME THAT IT IS NOT AT $T=0$ & CROSSES OVER TO FINITE T BEHAVIOUR.

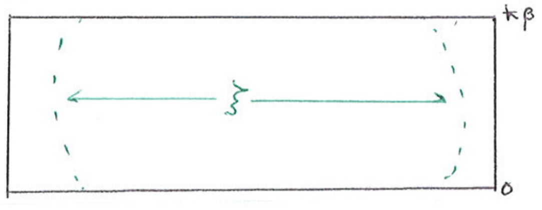


$|r-r_c| \ll 1$, $T \ll 1$ } $\hbar\omega \gg k_B T$

QUANTUM DISORDERED



$\frac{|r-r_c|^{\nu z}}{T} \sim 1$ } CROSSOVER REGIME



$|r-r_c| \ll 1$, $T \gg 1$ } $\hbar\omega \ll k_B T$

CORRELATION LENGTH IS SIZEABLE & $\xi_\tau \sim \xi^z$

CRITICAL BEHAVIOUR IS CLASSICAL QCR

CONT'D.

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- EXISTENCE OF A CROSSOVER FREQUENCY $k_BT/\hbar$ LEADS TO A PHYSICAL CROSSOVER THERMAL LENGTH $L_c^{1/z}$, WHICH THE SPATIAL FLUCTUATIONS ASSOCIATED WITH QUANTUM FLUCTUATIONS CANNOT EXCEED.

SEE N. GOLDENFELD "LECTURES ON PHASE TRANSITIONS & THE RENORMALIZATION GROUP" §9.11 FOR QUANTITATIVE INTRODUCTION TO FINITE SIZE SCALING.

BASICALLY IF f IS A FUNCTION WITH A HOMOGENEITY RELATION IN ~~THE~~ $L=\infty$ SYSTEM:

$$f_\infty(r,h) = b^{-d} f_\infty(\bar{r}\, b^{1/\nu}, \bar{h}\, b^{y_h})$$

$$f(r,h,L^{-1}) = b^{-d} f(\bar{r}\, b^{1/\nu}, \bar{h}\, b^{y_h}, b L^{-1})$$

L^{-1} BEHAVES LIKE A RELEVANT PERTURBATION WITH SCALING DIMENSION 1!

$\therefore$ WE CAN IMMEDIATELY WRITE DOWN THE FINITE TEMP. GENERALIZATION OF OUR SCALING FUNCTION:

$$\mathcal{O}(\vec{k},\omega,r,T) = \xi^{\dim[\mathcal{O}]}\, \Phi\left(k\xi,\ \omega\xi_z,\ \xi_z L_c^{-1}\right)$$

$$\xi \sim L_T^{1/z}, \quad \xi_z \sim L_z$$

$$\mathcal{O}(\vec{k},\omega,r,T) = \left(\frac{\hbar}{k_BT}\right)^{\dim[\mathcal{O}]/z} \Phi\left(\frac{\hbar k^z}{k_BT},\ \frac{\hbar\omega}{k_BT},\ \frac{\hbar|r-r_c|^{\nu z}}{k_BT}\right)$$

★ IMPORTANT POINT IS THAT TEMPERATURE ALONE SETS THE LENGTH SCALE ONE MUST MEASURE THE LENGTHSCALE AGAINST, THE TIME SCALE TO MEASURE THE FREQUENCY AGAINST & THE RELATIVE DISTANCE FROM THE CRITICAL POINT.

DIRECTLY AT THE CRITICAL COUPLING, $r=r_c$, T ALONE SETS THE ONLY ENERGY SCALE & MANY RESULTS WILL BE FULLY UNIVERSAL!

CONT'D

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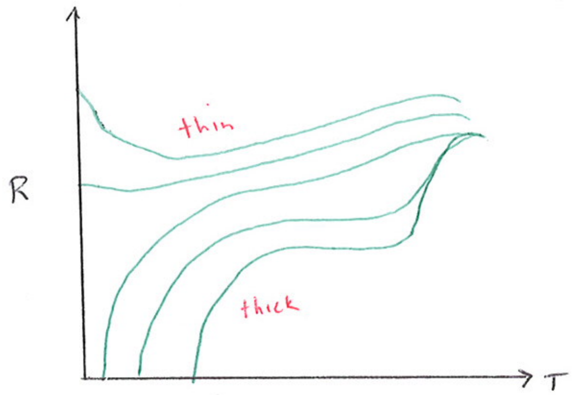
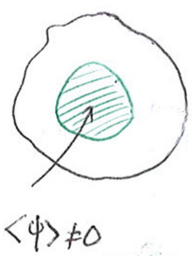
SUPPOSE WE HAVE TWO COMPETING THEORIES, LIKE FOR THE SC-M TRANSITION IN ULTRA NARROW WIRES.



THIS IS A QUANTUM PHASE TRANSITION BETWEEN A SUPERCONDUCTOR & SOMETHING ELSE. CAN WE USE THE SCALING THEORY HERE?

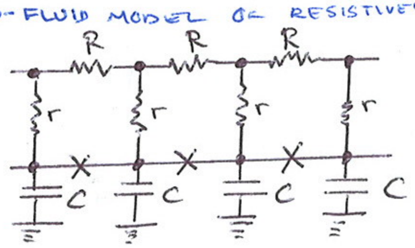
TWO COMPETING THEORIES, ONE FOR THE SC-M TRANSITION, ONE FOR THE SC-I TRANSITION. BASICALLY DIFFER IN THEIR MODEL SYSTEM

(i) A METALLIC WIRE WITH A STRONGLY SUPERCONDUCTING CORE



$$S = A\int_0^{\hbar\beta} d\tau \int_0^{\infty} dx \left\{ D|\partial_x \Psi|^2 + r|\Psi|^2 + \frac{u}{2}|\Psi|^4 \right\} + \frac{A}{\hbar\beta}\sum_{\omega_n}\int_0^{\infty} dx\, \gamma|\omega_n|\,|\Psi(x,\omega_n)|^2 \qquad \boxed{z=2}$$

(ii) TWO-FLUID MODEL OF RESISTIVELY COUPLED JOSEPHSON JUNCTIONS



$$S = \frac{A}{\hbar\beta}\sum_{\omega_n}\int\frac{dk}{2\pi}\left\{ \underbrace{\left(ck^2 + \frac{1}{c}\omega^2\right)\theta^2(k,\omega_n)}_{\text{plasmon}} + \underbrace{\gamma|\omega_n|\left(k^2 + R/r\right)\psi^2(k,\omega)}_{\text{dissipative}} + \cdots \right\} \qquad \boxed{z=1}$$

LET US TRY TO DISTINGUISH BETWEEN THESE THEORIES.

ON DIMENSIONAL GROUNDS ONLY WE KNOW:

$$[\sigma]_{3D} = \frac{S}{m} \qquad \text{FOR THIS CASE:} \qquad [A\sigma] = \frac{L}{\Omega} = \frac{e^2}{h}L \Rightarrow \dim[A\sigma] = 1$$

CON'TD.

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THE ONLY LENGTH SCALES WE HAVE ARE ξ & ξ_τ. ∴ USING THE DYNAMIC SCALING FORM:

$$\sigma_{id}(\vec{k},\omega,r,T) = \xi\,\Phi_\sigma\left(\frac{\hbar k^z}{k_BT},\ \frac{\hbar\omega}{k_BT},\ \frac{\hbar|r-r_c|^{\nu z}}{k_BT}\right)$$

AGAIN: $\xi \sim \xi_\tau^{1/z} = \left(\frac{\hbar}{k_BT}\right)^{1/z}$

$$\therefore\quad \sigma_{id}(\vec{k},\omega,r,T) = \left(\frac{\hbar}{k_BT}\right)^{1/z}\Phi_\sigma\left(\frac{\hbar k^z}{k_BT},\ \frac{\hbar\omega}{k_BT},\ \frac{\hbar|r-r_c|^{\nu z}}{k_BT}\right)$$

NOW, TAKE THE DC CONDUCTIVITY @ THE CRITICAL COUPLING: $k\to 0,\ \omega\to 0,\ r=r_c$

SOME UNIVERSAL #

$$\sigma_{id} \sim \frac{1}{T^{1/z}}\,\Phi(0)$$

CAN DISTINGUISH BETWEEN $\frac{1}{T}$ & $\frac{1}{\sqrt{T}}$ BEHAVIOUR IN THE EXPERIMENT!

PROBLEMS WITH THE QUANTUM CLASSICAL MAPPING

- MAY NOT HAVE STUDIED THE ASSOCIATED CLASSICAL FIELD THEORY (DISORDER)
- ALL CORRELATION FUNCTIONS GIVEN IN IMAGINARY TIME!
- ANALYTIC CONTINUATION & PERTURBATION THEORY DON'T MIX!